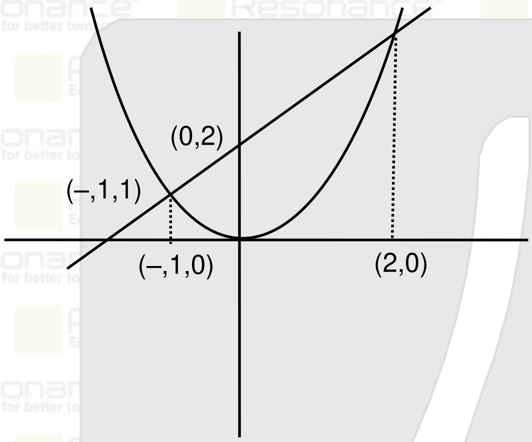
PART : MATHEMATICS

1 Area bounded by the curve $y = x^2$ and the line $y-x=2$ is

(1) $\frac{7}{2}$ (2) $\frac{9}{2}$ (3) $\frac{11}{2}$ (4) $\frac{13}{2}$

Ans. **(2)**

Sol.



$x^2-x-2 = 0$

$(x-2)(x+1) = 0$

Required area $= \frac{1}{2}(3)(5) - \int_{-1}^{2} x^2 dx$

$= \frac{15}{2} - \frac{1}{3}(8+1)$

$= \frac{15}{2} - 3 = \frac{9}{2}$ sq. unit

2. If $A_{3\times3} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + \ldots\ldots A^{20}$, then the sum of all the elements of M is :

(1) 1540 (2) 1580 (3) 2020 (4) 1640

Ans. **(3)**

Sol. $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$T_n = 1 + 2 + \ldots\ldots + n = \frac{n(n+1)}{2} = \frac{20\times21}{2} = 210$

$$S_n = \frac{1}{2}\left(\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right)$$

$$= \frac{n(n+1)}{4}\left(\frac{2n+1}{3} + 1\right)$$

$$= \frac{n(n+1)(n+2)}{6} = \frac{20\times21\times22}{6} = 1540$$

$$M = \begin{bmatrix} 1+1....20\text{times} & 1+2+3+.....20 & s_n \\ 0 & 1+1+1....20\text{times} & 1+2+3+....20 \\ 0 & 0 & 1+1+1....20\text{times} \end{bmatrix}$$

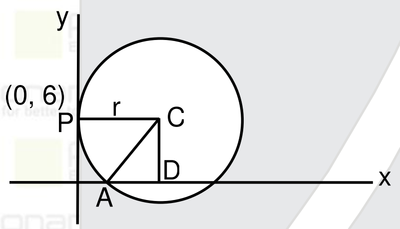
$$M = \begin{bmatrix} 20 & 210 & 1540 \\ 0 & 20 & 210 \\ 0 & 0 & 20 \end{bmatrix}$$

Sum of elements of M = 3(20) + 420 + 1540 = 2020

3. A circle touches y-axis at (0, 6) and has x-intercept equal to $6\sqrt{5}$ then radius of the circle is :

Ans. **9**

Sol.



AD = $3\sqrt{5}$

$CA^2 = CD^2 + AD^2$

= 36 + 45

$CA^2 = 81$

CA = 9

$\Rightarrow$ r = 9

4. Three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ with magnitude $\sqrt{2}$, 1 and 2 respectively follows the relation $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$.

The acute angle between the vectors $\vec{b}$ & $\vec{c}$ is θ. Then the value of (1+ tanθ) is :

Ans. **2**

Sol. $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$

$\vec{a} = (\vec{b}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{b})\vec{c}$

$\vec{a} = (|\vec{b}||\vec{c}|\cos\theta)\vec{b} - |\vec{b}|^2\vec{c}$

$\vec{a} = (2\cos\theta)\vec{b} - \vec{c}$

$\vec{a} \cdot \vec{b} = 2\cos\theta - 2\cos\theta$

$\Rightarrow \vec{a} \cdot \vec{b} = 0$

$\vec{a} \cdot \vec{a} = 2\cos\theta\,(\vec{b} \cdot \vec{a}) - \vec{c} \cdot \vec{a}$

$\vec{c} \cdot \vec{a} = -2$

$\vec{a} \cdot \vec{c} = 2\cos\theta\,(\vec{b} \cdot \vec{c}) - 4$

$-2 + 4 = 2\cos\theta \times 2\cos\theta$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 45^\circ$

So, $(1 + \tan\theta) = 2$

5. Two square matrices A & B are such that $A^2 + B^2$ is invertible and $A^5 = B^5$, $A^3B^2 = B^3A^2$. Then the value of $|A^3 - B^3|$ is :

Ans. **0**

Sol. $(A^3 - B^3)(A^2 + B^2) = A^5 - B^5 + A^3B^2 - B^3A^2 = 0$

Since $|A^2 + B^2| \neq 0 \Rightarrow |A^3 - B^3| = 0$

6. Evaluate $\lim\limits_{x\to 0}\frac{x}{\sqrt[8]{1-\sin x} - \sqrt[8]{1+\sin x}}$

(1) 0 (2) – 4 (3) 8 (4) 2

Ans. **(2)**

Sol. Rationalize denominator three times

$$\lim_{x\to 0}\frac{x\left\{(1-\sin x)^{1/8} + (1+\sin x)^{1/8}\right\}\left\{(1-\sin x)^{1/4} + (1+\sin x)^{1/4}\right\}\left\{(1-\sin x)^{1/2} + (1+\sin x)^{1/2}\right\}}{(1-\sin x - 1 - \sin x)}$$

$$\lim_{x\to 0}\frac{8x}{-2\sin x}$$

$= -4$

7. If two given APs are $\tan\frac{\pi}{9}$, x, $\tan\frac{7\pi}{18}$ and $\tan\frac{\pi}{9}$, y, $\tan\frac{5\pi}{18}$. Then the value of $|x-2y|$ is :

Ans. **0**

Sol. $\left|\frac{1}{2}\left(\tan\frac{\pi}{9} + \tan\frac{7\pi}{18}\right) - \tan\frac{\pi}{9} - \tan\frac{5\pi}{18}\right|$

$\because 2x = \tan\frac{\pi}{9} + \tan\frac{7\pi}{18}$ and $2y = \tan\frac{\pi}{9} + \tan\frac{5\pi}{18}$

$\left|\frac{1}{2}\left(\cot\frac{\pi}{9}-\tan\frac{\pi}{9}\right)-\tan\frac{5\pi}{18}\right|$

$\left|\frac{1}{2}\times 2\cot\frac{2\pi}{9}-\tan\frac{5\pi}{18}\right|$

$|\cot 40° - \tan 50°| = 0$

8. Let f(x) be a twice differential function defined as $f(x) = \int_a^x g(t)dt$ and f(x) lies between a and b. If f(x) has 5 roots, then the minimum number of roots of the equation g'(x) g(x) = 0 is :

(1) 12 (2) 3 (3) 5 (4) 7

Ans. **(4)**

Sol. f'(x) = g(x)

g'(x) = f''(x)

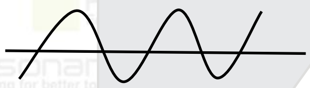
$f(x) = \int_a^x g(t)dt$

g(x) = f'(x)

g'(x) = f''(x)

g(x) . g'(x) = f'(x) f''(x)

Graph of f(x) can be drawn as



If f(x) = 0 has atleast 5 real roots

f'(x) = 0 has atleast 4 real root

f''(x) = 0 has atleast 3 real root

f'(x) f''(x) = 0 has atleast 7 real root

g(x) g'(x) = 0 has atleast 7 real root

9. If $(x + x^3)\,dy = (y + yx^2 + x^3)dx$ and y(1) = 0, then y(2) is :

(1) $\ln\left(\frac{17}{2}\right)$ (2) 0 (3) $\ln\left(\frac{5}{2}\right)$ (4) $\ln\left(\frac{2}{5}\right)$

Ans. **(3)**

Sol. $\frac{dy}{dx}=\frac{y+yx^2+x^3}{x+x^3}$

$\frac{dy}{dx}-\frac{y}{x}=\frac{x^2}{x^2+1}$

I.F = $e^{-\int \frac{1}{x}dx} = \frac{1}{x}$

$y\left(\frac{1}{x}\right) = \int \frac{x^2}{x^2+1}\left(\frac{1}{x}\right)dx$

$\Rightarrow \frac{y}{x} = \frac{1}{2}\ln(1+x^2) + \ln c$

$\Rightarrow y = \frac{x}{2}(\ln(1+x^2) + \ln c)$

Now y(1) = 0

$\Rightarrow 0 = \frac{1}{2}(\ln 2 + \ln c)$

ln 2c = 0

$\Rightarrow c = \frac{1}{2}$

Now y(2) = $\frac{2}{2}(\ln 5 + \ln \frac{1}{2})$

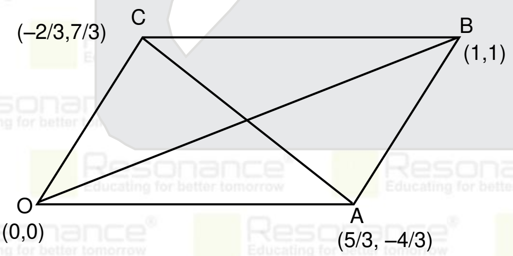
$y(2) = \ln\frac{5}{2}$

10. If two sides of a parallelogram are 4x + 5y = 0 and 7x + 2y = 0 . If one of the diagonals is 11x + 7y = 9 then the other diagonal passes through the point :

(1) $\left(\frac{1}{3}, -\frac{4}{3}\right)$ (2) (1,1) (3) (2,3) (4) $\left(\frac{2}{3}, \frac{4}{3}\right)$

Ans. **(2)**

Sol. AC : 11x + 7y = 9



OA : 4x + 5y = 0

OC : 7x + 2y = 0

$\Rightarrow A\left(\frac{5}{3}, \frac{-4}{3}\right)$ and $C\left(\frac{-2}{3}, \frac{7}{3}\right) \Rightarrow M = \left(\frac{1}{2}, \frac{1}{2}\right)$

equation of OB $\Rightarrow$ y = x

11. The point P(a, b) undergoes following transformation to a new co-ordinate P' $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

(i) Reflection about y = x

(ii) Translation through 2 units in the positive direction of x-axis

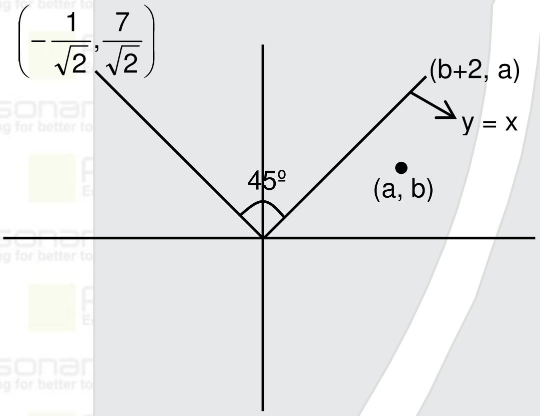
(iii) Rotation through an angle $\frac{\pi}{4}$ in anti clockwise sense about the origin

Then the value of 2a + b is :

(1) 1 (2) 3 (3) 4 (4) 9

Ans. (4)

Sol.



Using rotation theorem

$\frac{-1}{\sqrt{2}} + \frac{7}{\sqrt{2}}i = \{(b + 2) + ai\} \left\{\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right\}$

$\frac{-1}{\sqrt{2}} + \frac{7}{\sqrt{2}}i = \left(\frac{b+2}{\sqrt{2}} - \frac{a}{\sqrt{2}}\right) + i\left(\frac{b+2}{\sqrt{2}} + \frac{a}{\sqrt{2}}\right)$

$b - a + 2 = -1$ .... (i)

$b + 2 + a = 7$ ...... (ii)

Using equation (i) & (ii)

a = 4, b = 1

$\Rightarrow 2a + b = 9$

12. If $\alpha^{1/5}$ and $\beta^{1/5}$ are roots of the equation $8x^2 + bx + c = 0$ where $\alpha = \max(2^{6\sin 3x + 8\cos 3x})$ and $\beta = \min(2^{6\sin 3x + 8\cos 3x})$. Then the value of |b–c| is :

Ans. **(42.0)**

Sol. $\alpha = \max(2^{6\sin 3x + 8\cos 3x}) = 2^{10}$

$\beta = \min(2^{6\sin 3x + 8\cos 3x}) = 2^{-10}$

$\alpha^{1/5} = 2^2$ and $\beta^{1/5} = 2^{-2}$

$\Rightarrow 8x^2 + bx + c = 0 \begin{cases} \nearrow \alpha^{1/5} \\ \searrow \beta^{1/5} \end{cases}$

$\Rightarrow \alpha^{1/5} + \beta^{1/5} = 4 + \frac{1}{4} = \frac{17}{4} = \frac{-b}{8}$

$b = -34$

$= \alpha^{1/5} + \beta^{1/5} = 1 = \frac{c}{8} \Rightarrow c = 8$

$|b - c| = 42$

13. For an ellipse E, centre lies at the point (3, – 4), one of the foci is at (4, – 4) & one of vertices is at (5, – 4). If the equation of tangent on the ellipse E is $mx - y = 4 (m > 0)$. The value of $5m^2$ is :

(1) 3 (2) 5 (3) 9 (4) 7

Ans. **(1)**

Sol. Equation of the ellipse is $\frac{(x-3)^2}{a^2} + \frac{(y+4)^2}{b^2} = 1$

$\frac{4}{a^2} = 1 \Rightarrow a^2 = 4$

$a - ae = 1$

$2 - 2e = 1 \Rightarrow e = \frac{1}{2}$

$b^2 = a^2 - a^2 e^2 = 4 - 4\left(\frac{1}{4}\right) = 3$

$\Rightarrow \frac{(x-3)^2}{4} + \frac{(y+4)^2}{3} = 1$

Tangent $\rightarrow y + 4 = m(x - 3) \pm \sqrt{4m^2 + 3}$

Given tangent is $y = mx - 4$

$3m = \pm\sqrt{4m^2 + 3} \Rightarrow 9m^2 = 4m^2 + 3$

$5m^2 = 3$

14. The number of solutions of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is :

Ans. **(2)**

Sol. $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$

$e^{2x} - e^x - 4 - \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$

$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 - \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - \left(e^x + \frac{1}{e^x}\right) - 6 = 0$

Put $e^x + \frac{1}{e^x} = t$

$\Rightarrow t^2 - t - 6 = 0$

$t = 3, t = -2$ (not possible)

$\Rightarrow e^x + \frac{1}{e^x} = 3$

$e^{2x} - 3e^x + 1 = 0$

Put $e^x = p$

$\Rightarrow p^2 - 3p + 1 = 0$

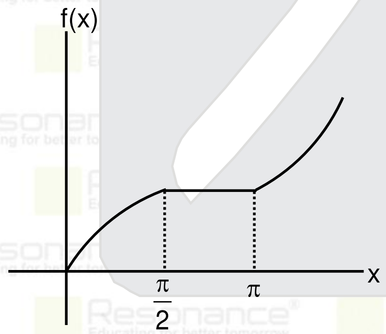
$\Rightarrow$ number of solutions = 2

15. If $f(x) = \begin{cases} \max(\sin t) & ; \quad 0 \le t \le x,\ x \in [0, \pi] \\ 2 + \cos x & ; \quad x > \pi \end{cases}$ then the number of points where of f(x) is not continuous or non-differentiable.

(1) 0 (2) 1 (3) 2 (4) infinite

Ans. **(1)**

Sol.



Critical point $\frac{\pi}{2}$ & π

At $x = \frac{\pi}{2}$

f (x) is continuous and differentiable at $x = \frac{\pi}{2}$ $\Rightarrow$ LHD = RHD = 0

at $x = \pi$

f(x) is again continuous and differentiable at $x = \pi$ $\Rightarrow$ LHD = RHD = 0

So, number of dis-continuous and non differentiable point = 0

16. If $f(x)=\dfrac{1-\sin x+\cos x}{1+\sin x+\cos x}, x\neq\pi$ then the value of $f(\pi)$ so that f(x) is continuous.

(1) 1 (2) 0 (3) –1 (4) 2

Ans. **(3)**

Sol. Since f(x) is continuous at $x = \pi$

$$\Rightarrow f(\pi)=\lim_{x\to\pi}\frac{1-\sin x+\cos x}{1+\sin x+\cos x}$$

$$=\lim_{x\to\pi}\frac{2\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$=\lim_{x\to\pi}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}=\frac{0-1}{0+1}=-1$$

So $f(\pi) = -1$

17. Let $\left(3^{\frac{1}{6}\log_3(25^{x-1}+7)}+3^{\frac{1}{8}\log_3(5^{x-1}+1)}\right)^{10}$ is expanded in the increasing power of $3^{\frac{1}{8}\log_3(5^{x-1}+1)}$. If 9th terms of the expansion is 180 then the value of x is :

(1) –1 (2) 1 (3) 0 (4) 2

Ans. **(2)**

Sol. 9th terms of the expansion $\left((25^{x-1)}+7)^{\frac{1}{6}}+(5^{x-1}+1)^{\frac{1}{8}}\right)^{10}$ is

$${}^{10}C_8\left((25^{x-1)}+7)^{\frac{2}{6}}\times(5^{x-1}+1)^{\frac{8}{8}}\right)=180$$

$$\Rightarrow\frac{10\times9}{2}\left((25^{x-1)}+7)^{\frac{2}{3}}\times(5^{x-1}+1)\right)=180$$

After solving x :

x = 1